Analytic elements for flow in layered, anisotropic aquifers

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The Girinski potential for groundwater flow in stratified aquifers has been extended to allow for a different anisotropic horizontal hydraulic conductivity in each layer. The difference in anisotropy between layers creates a three–dimensional flow field, which may become complicated, for example near wells.

1. Introduction

The theory for Dupuit groundwater flow in stratified aquifers consisting of a number of homogeneous layers, based on the Girinski potential [1,2], is well known and has been implemented in several analytic element programs (SLWL [2]; TwoDan [3]). Recently, this theory was extended to allow for a different horizontal anisotropy in each layer [4] as long as the flow remains confined. The essential consequence of a different anisotropy in each layer is that, in general, this will result in a three–dimensional flow field, even when all analytic elements (such as a well or a line–sink) penetrate the aquifer fully.

2. Solution method and results

Equations for the head as a function of the horizontal coordinates may be derived in a scaled coordinate system by using a comprehensive transmissivity tensor; this procedure is not any different from that for aquifers with only one anisotropic horizontal hydraulic conductivity (e.g. [2], p. 211). Equations for the horizontal flow in each layer may be obtained by application of Darcy's law. And finally, the vertical component of flow at the layer interfaces may be obtained through vertical integration of the horizontal divergence of each layer; the vertical component of flow varies linearly within each layer and is continuous at the interfaces. Equations for the head and the horizontal and vertical flow components for uniform flow and flow to a well in an aquifer consisting of an arbitrary number of layers are given in [4]. The solutions may be superimposed to solve more complicated problems.

The three–dimensional flow field in layered, anisotropic aquifers can become quite complicated, especially near pumping wells. In the context of transport modeling the three–dimensional character of the flow lines can be of great importance. In a two–layer system, the flow lines to a well may be grouped into four bundles of spiraling flow lines bounded

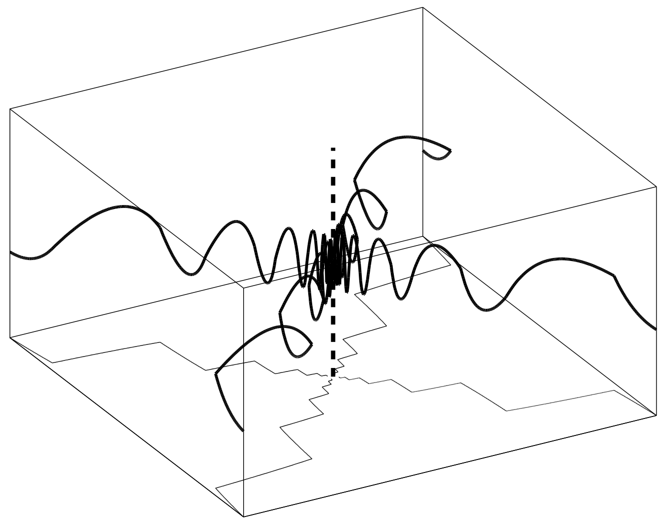

Figure 1. Four spiraling flow lines towards a well in a two–layer system. Thin lines are projection on horizontal plane, dashed line is well screen. Vertical scale exaggerated.

by two vertical planes that intersect at the well. The bundles of spiraling flow lines were first observed from numerical experiments of flow in anisotropic multiaquifer systems by [5] and are referred to as groundwater whirls. An example of flow lines to a well in a two–layer aquifer is shown in Figure 1. The major principal directions of the horizontal hydraulic conductivities of the two layers are normal to each other, but both the major and minor values are equal. The thickness of the two layers is equal and thus the comprehensive transmissivity vector is isotropic. The thick lines are flow lines to the well in the center; the thin lines are projections on a horizontal plane; the thick dashed line is the well screen.

The presentation will outline the new Dupuit theory for analytic elements in layered anisotropic aquifers and will show a number of examples of three–dimensional flow fields that can be obtained through superposition of uniform flow, wells, and circular infiltration areas. Both two–layer and three–layer systems will be discussed.

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